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TRANSIENT RESPONSE OF LAMINATED, BIMODULAR-MATERIAL, COMPOSITE --ETC(U)
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TRANSIENT RESPONSE OF LAMINATED, BIMODULAR-MATERIAL, COMPOSITE RECTANGULAR PLATES

by

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TRANSIENT RESPONSE OF LAMINATED, BIMODULAR-MATERIAL, COMPOSITE RECTANGULAR PLATES

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The paper presents a finite-element analysis of the transient behavior of fiber-reinforced, single-layer and two-layer cross-ply rectangular plates of bimodular materials (i.e., materials whose linear elastic properties are different depending on whether the fiber-direction strains are tensile or compressive). To validate the finite element results, a closed-form solution is also presented for a rectangular plate with all edges simply-supported without in-plane restraint (along the edge) and tangential rotation and subjected to suddenly applied sinusoidally distributed normal pressure. The time behavior of the transverse loading is arbitrary (e.g., step loading, impulse loading, etc.). Numerical results for transverse deflection and locations of the 'neutral surface' as functions of time are presented for two bimodular materials. The finite element results agree very closely with the closed-form solutions.

Introduction

In 1941 S. Timoshenko [1] considered the flexural stresses in materials which have different moduli in tension and in compression. Examples of such materials, called bimodular materials, are provided by cord-rubber composites, certain biological tissues, and paperboard, among others. Analysis of bimodular (or bimodulus) materials in two dimensions began with the work of Ambartsumyan [2] in 1965. Following this work, there appeared numerous static, two-dimensional analyses of bimodular materials. Among these, the works of Kamiya [3-5], Jones and Morgan [6], and Bert, Reddy and their colleagues [7-11] should be particularly noted in the context of bending of bimodular plates. For additional references to the subject, the reader is referred to the bibliography in [7-11]. To the best of the present author's knowledge,

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no previous transient analyses of plates laminated of bimodular composite materials are available in the open literature. The present work is believed to be the first one to consider the transient analysis of layered composite plates of bimodular materials.

The present study employs the finite element model developed in [9] with the Nemark direct integration technique [12]. The closed form solution presented herein is essentially an extension of the static solution presented in [11] to transient solution. The finite element model and the closed-form solution are based on a shear-deformation theory (see Whitney and Pagano [13]) of layered composite plates, which does not account for delamination between layers.

Theory and Formulation

The displacement field in all of the simple Timoshenko-type shear-deformation theories are based on the following displacement field:

$$\begin{aligned} u_1(x,y,z,t) &= u(x,y,t) + z\phi_x(x,y,t) \\ u_2(x,y,z,t) &= v(x,y,t) + z\phi_y(x,y,t) \\ u_3(x,y,z,t) &= w(x,y,t). \end{aligned} \tag{1}$$

Here t is the time, u_i $(i = 1,2,3)$ is the displacement in x_i-coordinate direction $(x_1 = x,\ x_2 = y,\ x_3 = z)$, (u,v,w) are the associated midplane displacements, and ϕ_x and ϕ_y are the bending slopes in the xz and yz planes. The coordinate system is chosen such that the xy-plane coincides with the midplane of the plate.

The equations of motion in the small-deflection theory of moderately thick plates are given by

$$\begin{aligned} N_{1,x} + N_{6,y} &= Pu_{,tt} + R\phi_{x,tt} \\ N_{6,x} + N_{2,y} &= Pv_{,tt} + R\phi_{y,tt} \\ Q_{1,x} + Q_{2,y} &= Pw_{,tt} + q(x,y,t) \end{aligned} \tag{2}$$

$$M_{1,x} + M_{6,y} - Q_1 = I\phi_{x,tt} + Ru_{,tt}$$

$$M_{6,x} + M_{2,y} - Q_2 = I\phi_{y,tt} + Rv_{,tt}$$

where P, R and I are the normal, coupled normal-rotary, and rotary inertia coefficients,

$$(P,R,I) = \int_{-h/2}^{h/2} (1,z,z^2)\rho dz = \sum_m \int_{z_m}^{z_{m+1}} (1,z,z^2)\rho^{(m)} dz \qquad (3)$$

$\rho^{(m)}$ being the material density of the m-th layer, N_i, Q_i, and M_i are the stress and moment resultants.

The main difference between the usual plate theory and the laminated plate theory is reflected in the plate constitutive equations. For bimodular-material plates these constitutive equations are given by

$$\begin{Bmatrix} N_i \\ M_i \end{Bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} \\ B_{ji} & D_{ij} \end{bmatrix} \begin{Bmatrix} \varepsilon_j \\ K_j \end{Bmatrix}, \quad (i,j = 1,2,6) \qquad (4)$$

$$\begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} = \begin{bmatrix} k_4^2 S_{44} & k_4 k_5 S_{45} \\ k_4 k_5 S_{45} & k_5^2 S_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_4 \\ \varepsilon_5 \end{Bmatrix}. \qquad (5)$$

Here, A_{ij}, B_{ij}, D_{ij}, and S_{ij} denote the respective in-plane, bending-stretching coupling, bending or twisting, and thickness-shear stiffnesses defined as follows:

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} (1,z,z^2) Q_{ijk\ell} dz \quad (i,j = 1,2,6)$$

$$S_{ij} = \int_{-h/2}^{h/2} Q_{ijk\ell}\, dz \quad (i,j = 4,5) \qquad (6)$$

where h is the total thickness of the plate, $Q_{ijk\ell}$ denotes the plane-stress reduced stiffness (i,j refer to the position in the compliance matrix; k refers to the sign of the fiber-direction strain: k = 1, tensile and k = 2, compressive; and ℓ refers to the layer number), ε_j and K_j are the strains and curvatures associated with the displacements in (1), and k_i are the shear convection coefficients.

Closed-Form Solutions

In [8,10,11] it was shown that the closed-form solution can be derived for a freely supported, lamianted, bimodular, rectangular plate subjected to a sinusoidally distributed normal pressure. Guided by these results, we consider the same problem but subjected to sinusoidal distributed, time-dependent load,

$$q(x,y,t) = (q_0 \sin \alpha x \sin \beta y)T(t), \quad \alpha = \pi/a, \quad \beta = \pi/b \tag{7}$$

where T(t) is a known function of time, and a and b are the planform dimensions of the plate. The boundary condtions are given by

$$\begin{aligned} &\text{at } x = 0,a: \quad v = w = \phi_y = N_1 = M_1 = 0 \\ &\text{at } y = o,b: \quad u = w = \phi_x = N_2 = M_2 = 0. \end{aligned} \tag{8}$$

For the loading and boundary conditions given above, the governing equations (2) are satisfied exactly (for any t > 0) by the following form of the generalized displacements:

$$\begin{aligned} u(x,y,t) &= U(t)\phi_1(x,y) \\ v(x,y,t) &= V(t)\phi_2(x,y) \\ w(x,y,t) &= W(t)\phi_3(x,y) \\ \phi_x(x,y,t) &= X(t)\phi_1(x,y) \\ \phi_y(x,y,t) &= Y(t)\phi_2(x,y) \end{aligned} \tag{9}$$

where

$$\phi_1 = \cos \alpha x \sin \beta y, \ \phi_2 = \sin \alpha x \cos \beta y, \ \phi_3 = \sin \alpha x \sin \beta y. \quad (10)$$

The values of the coefficients U(t), V(t), W(t), X(t) and Y(t) are obtained by solving the following ordinary differential equations in time:

$$[M] \begin{Bmatrix} \ddot{U} \\ \ddot{V} \\ \ddot{W} \\ \ddot{X} \\ \ddot{Y} \end{Bmatrix} + [C] \begin{Bmatrix} U \\ V \\ W \\ X \\ Y \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ T \\ 0 \\ 0 \end{Bmatrix} \quad (11)$$

The elements of [C] and [M] are given in [14].

It should be pointed out that the plate stiffnesses A_{ij}, B_{ij}, and D_{ij} depend on the neutral-surface positions Z_{nx} and Z_{ny} associated with the fiber-direction strains ε_x and ε_y, respectively. The neutral-surface locations are given by,

$$Z_{nx}(t) = -U(t)/X(t)$$
$$Z_{ny}(t) = -V(t)/Y(t). \quad (12)$$

which are constants for any fixed time. Thus, for any $t > 0$, the neutral surfaces are planes, as in the case of static bending. For the detailed computations of A_{ij}, B_{ij}, and D_{ij}, the reader is referred to [8,9,11].

Next, we consider the numerical solution of (11) by the Newmark direct integration technique. Equation (11) is of the general form

$$[M]\{\ddot{\Delta}\} + [K]\{\Delta\} = \{F\} \quad (13)$$

where $\{\Delta\}$ denotes the column of the generalized displacements. In Newmark's method, the solution and its derivative with respect to time are approximated by

$$\{\Delta\}_{n+1} = \{\Delta\}_n + \Delta t\{\dot{\Delta}\}_n + [(\tfrac{1}{2} - \beta)\{\ddot{\Delta}\}_n + \beta\{\ddot{\Delta}\}_{n+1}](\Delta t)^2$$

$$\{\dot{\Delta}\}_{n+1} = \{\dot{\Delta}\}_n + [(1 - \alpha)\{\ddot{\Delta}\}_n + \alpha\{\ddot{\Delta}\}_{n+1}]\Delta t \qquad (14)$$

where α and β are the parameters that control the stability of the scheme, $\{\Delta\}_n$ denotes the value of $\{\Delta\}$ at time $t = n\ \Delta t$. The choice $\alpha = 0.5$ and $\beta = 0.25$ is known to give an unconditionally stable scheme for linear problems.

Using of the approximations (14) in (11) and rearranging the terms, we arrive at

$$[\hat{K}]\{\Delta\}_{n+1} = \{\hat{F}\}_{n,n+1} \qquad (15)$$

where

$$[\hat{K}] = [K] + a_0[M]$$

$$\{\hat{F}\} = \{F\}_{n+1} + [M](a_0\{\Delta\}_n + a_1\{\dot{\Delta}\}_n + a_2\{\ddot{\Delta}\}_n) \qquad (16)$$

$$a_0 = 1/(\beta\Delta t) \quad , \quad a_1 = a_0\Delta t \quad , \quad a_2 = \frac{1}{2\beta} - 1.$$

Starting with initial values of $\{\Delta\}_0$, $\{\dot{\Delta}\}_0$ and $\{\ddot{\Delta}\}_0$, equation (15) can be solved repeatedly for $\{\Delta\}$ at successive values of time. The values of $\{\dot{\Delta}\}_{n+1}$ and $\{\ddot{\Delta}\}_{n+1}$ can be computed from (14).

Finite-Element Formulation

As pointed out earlier, the finite element model used in the present study is the same as that employed in [9]. We shall not repeat the formulation here, but only point out the additional steps involved in the transient analysis. The finite-element model in the present case results in the following equation for an element

$$[M]\{\ddot{\Delta}\} + [\bar{K}]\{\Delta\} = \{F\}, \qquad (17)$$

where $\{\Delta\}$ denotes the column of the nodal values of the generalized displacements. The elements of the mass matrix [M] and the stiffness

matrix $[\overline{K}]$ are given in [14,15]. Equation (17) is integrated using the direct integration technique described above.

It should be noted that no restriction is placed on the loading and boundary conditions in the finite-element analysis. When a uniform loading is used, the neutral-surface locations are not independent of the location (x,y), and therefore, the expressions for A_{ij}, B_{ij} and D_{ij} derived for the closed-form solutions are not valid in the entire plate in the finite-element analysis. Since the coefficients $\overline{K}_{ij}$ are evaluated at the Gauss points, the stiffnesses A_{ij}, B_{ij} and D_{ij} are also evaluated at the Gauss points by using the neutral-surface positions computed at the Gauss points. This is equivalent to the assumption that the neutral surfaces are element-wise bilinear.

In the actual calculation of the stiffnesses, whether for the closed-form solution or in the finite-element analysis, the neutral-surface locations are not known a-priori. Therefore an iterative technique must be employed to compute the neutral-surface locations. The iteration procedure begins with assumed values of Z_{nx} and Z_{ny} (say, $Z_{nx} = Z_{ny} = 0$) and then the stiffnesses A_{ij} etc. are computed using these values. After obtaining the generalized displacements, the neutral-surface locations are recomputed. Using these new values of Z_{nx} and Z_{ny}, the stiffnesses for the next iteration are computed. This procedure is repeated until the difference between any two consecutive values of Z_{nx} (and Z_{ny}) differ by a small preselected value (say 0.1%).

Numerical Results and Discussion

In the following, numerical results are presented for rectangular plates made of two bimodular materials: aramid cord-rubber (AR) and polyester cord-rubber (PR), which are used in automobile tires. The

material properties for these two materials are given in [7-11,14], and are listed in Table 1 again for convenience. In the finite element method, a 2 x 2 mesh of nine-node isoparametric elements in the quarter plate was used. The shear correction coefficients k_i^2 were chosen to be 5/6.

Table 1 Material properties for aramid cord-rubber and polyester cord-rubber, unidirectional, bimodulus composite materials.

Property	Aramid-Rubber		Polyester Rubber	
	Tensile	Compressive	Tensile	Compressive
E_{11} (GPa)	3.58	0.012	0.617	0.0369
E_{22} (GPa)	0.00909	0.012	0.008	0.0106
ν_{12}	0.416	0.205	0.475	0.185
$G_{12} = G_{13}$ (GPa)	0.0037	0.0037	0.00262	0.00267
G_{23} (GPa)	0.0029	0.00499	0.00233	0.00475
ρ (GPa)	1.0	1.0	1.0	1.0

Due to the lack of other results in the literature, comparisons are made between the present closed-form solution (CFS) and finite element solutions (FES). The selection of the time step was guided by the stability criterion given by Tsui and Tong [16] for moderately thick plates (see [15]).

First, the effect of the time step on the transverse deflection and the neutral-surface location of a single-layer (0°) square plate (a = b = 1, a/h = 5, aramid-rubber) under suddenly applied step loading (with $q_0 = 1$ in eqn. (7)) was studied. Table 2 contains the results for the transverse deflection and the neutral-surface location for various time steps. Note that the solutions obtained using $\Delta t = 0.5$ differ from those obtained using $\Delta t = 0.05$ by only 2%.

Next, the finite-element results are validated by comparing with the closed-form solutions of a two-layer (0°/90°) square plate (a = b = 1, a/h = 5, aramid-rubber) under suddenly applied step loading (7).

Table 2 Effect of the time step on the transverse deflection and the neutral-surface location of a single-layer (aramid-rubber) square plate under suddenly applied, sinusoidally distributed step loading (closed-form solution).

Δt	0.05		0.1		0.2		0.25		0.5	
t	$w \times 10^{-4}$	Z_x/h	$w \times 10^{-4}$	Z_x/h	$w \times 10^{-4}$	Z_x/h	$w \times 10^{-4}$	Z_x/h	$w \times 10^{-4}$	Z_x/h
1.0	0.00024	0.1258	0.00023	0.1246	0.00020	0.1307	0.00019	0.1373	0.000151	0.1813
2.0	0.00096	0.2754	0.00093	0.2737	0.00089	0.2714	0.00086	0.2706	0.000767	0.2713
4.0	0.00370	0.3837	0.00366	0.3829	0.00357	0.3815	0.00353	0.3809	0.00332	0.3777
6.0	0.00796	0.4246	0.00790	0.4243	0.00778	0.4236	0.00772	0.4233	0.00742	0.4213
8.0	0.0135	0.4397	0.0134	0.4396	0.0133	0.4394	0.0132	0.4393	0.0128	0.4286
10.0	0.0200	0.4421	0.0199	0.4421	0.0198	0.4422	0.0197	0.4422	0.0192	0.4422
11.0	0.0235	0.4415	0.0234	0.4415	0.0232	0.4416	0.0231	0.4416	0.0227	0.4418
12.0	0.0270	0.4408	0.0269	0.4408	0.0267	0.4409	0.0266	0.4409	0.0262	0.4411
14.0	0.0340	0.4413	0.0339	0.4413	0.0337	0.4413	0.0336	0.4412	0.0332	0.4411
15.0	0.0373	0.4429	0.0372	0.4427	0.0371	0.4426	0.0370	0.4426	0.0366	0.4423

Table 3 contains the results for the transverse deflection, transverse velocity, and the neutral surface location, z_{nx}. The finite element results are in excellent agreement (less than one percent error) with the closed-form solution. Both the closed-form and finite-element solutions were obtained using $\Delta t = 0.5$, and $q_0 = 1$ in eqn. (7). In view of the close agreement between the closed-form and finite-element solutions, and in the interest of saving the computational time (the finite element method took 30 min. of CPU time compared to 5 sec. for the closed form solution presented in Table 3), most of the results to be discussed were obtained using the closed-form method. Of course, for the uniformly distributed load case, the finite element method was employed.

Figure 1 contains the plots of the transverse deflection and the neutral-surface location versus time for a single-layer (0°), square plate ($a = b = 1$, $a/h = 5$, aramid-rubber) subjected to suddenly applied step loading ($q_0 = 1$, $\Delta t = 0.5$). The figure shows the effect of the shear deformation and the bimodular action on the amplitude and period of the solutions. Clearly, the effect of both the shear deformation and the bimodular action is very pronounced and hence cannot be neglected in the analysis. It should be pointed out that the value of Z_{nx}/h for the time interval $8 < t < 36$, coincides with the Z_{nx}/h of the associated static case (see [11]).

To further investigate the effect of the plate thickness on the dynamic response, the same problem (as in Figure 1) was solved using $a/h = 10$. The nondimensionalized transverse deflection, $\bar{w}=(wh^3E^c_{22})/q\ a^4_0$, versus time is shown in Fig. 2. The effect of the thickness on the amplitude and period of the deflection is apparent from the figure.

Table 3 Comparison of the neutral-surface location, transverse deflection and transverse velocity as obtained by the closed-form solution (CFS) and the finite element method (FEM) for a two-layer (0°/90°) square plate (aramid-rubber) under suddenly applied transverse step loading†.

t	Z_{nx}/h		$w \times 10^{-3}$		$\dot{w} \times 10^{-2}$	
	CFS	FEM	CFS	FEM	CFS	FEM
4	0.3780	0.3804	0.0330	0.0330	0.1674	0.1671
8	0.4387	0.4385	0.1277	0.1277	0.2959	0.2966
12	0.4377	0.4370	0.2597	0.2596	0.3486	0.3477
16	0.4395	0.4396	0.3904	0.3900	0.2908	0.2902
20	0.4410	0.4402	0.4832	0.4826	0.1614	0.1603
24	0.4388	0.4389	0.5127	0.5112	-.0208	-.0223
28	0.4419	0.4411	0.4684	0.4665	-.1912	-.1920
32	0.4373	0.4372	0.3656	0.3632	-.3116	-.3136
36	0.4412	0.4410	0.2307	0.2284	-.3457	-.3438
40	0.4279	0.4255	0.1044	0.1026	-.2745	-.2735

† $a/b = 1$, $a/h = 5$, $\Delta t = 0.5$, $\rho = 1.0$, $q_o = 1.0$

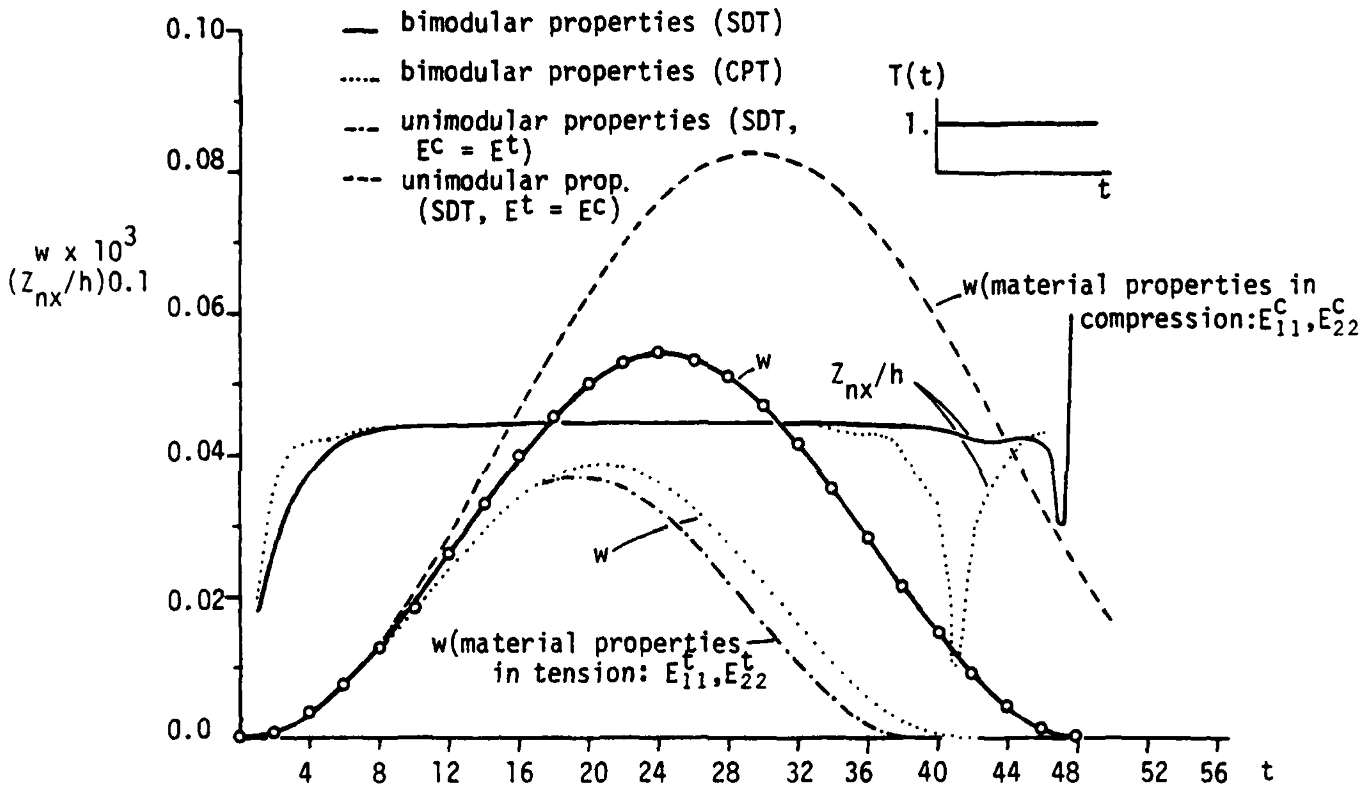

Figure 1. Effect of the transverse shear deformation and bimodulus action on the transient response of single-layer (0^o) square plates ($a = b = 1$, $h = 0.2$, $q_o = 1$, $\Delta t = 0.5$) of aramid-rubber under suddenly applied, sinusoidally distributed, step loading.

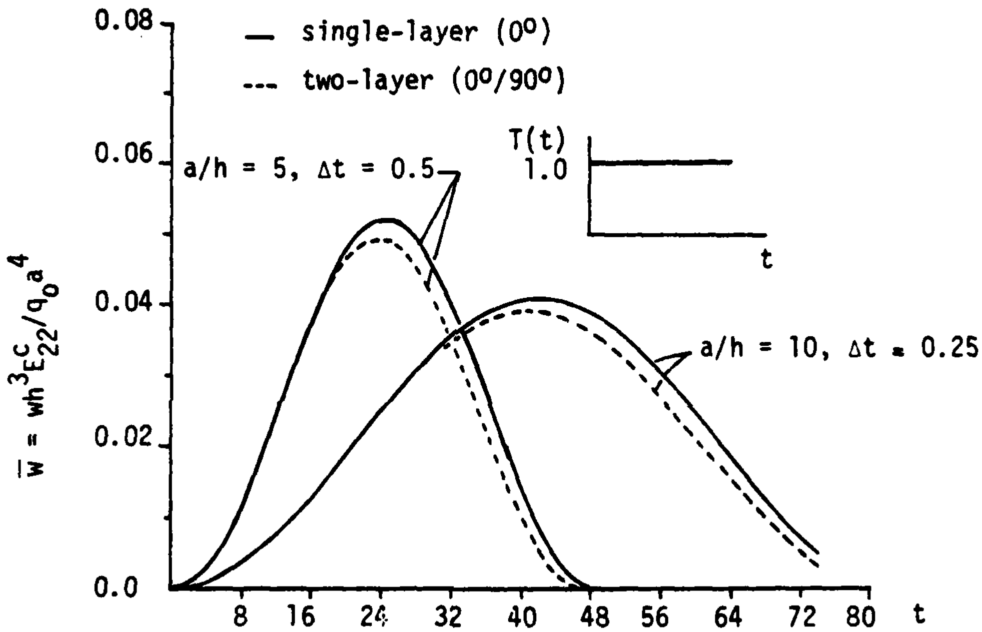

Figure 2. Effect of the plate thickness on the transient response of single-layer and two-layer, bimodular, square plates of aramid-rubber material under suddenly applied, sinusoidally distributed, step loading.

The transverse deflection, transverse velocity, and the neutral-surface location versus time for a single-layer (0°), square plate of polyester-rubber material ($a/h = 5$, $q_0 = 1$, $\Delta t = 0.5$) under suddenly applied step loading are shown in Fig. 3. The figure also contains the plot of the transverse deflection versus time for a rectangular plate ($a/b = 2$; everything else is the same as in the square plate).

In Fig. 4, results of two-layer (0°/90°) square plates ($a = b = 1$, $a/h = 5$, $\Delta t = 0.5$, $q_0 = 1$) of aramid-rubber and polyester-rubber are presented. Once again, the excellent agreement between the closed-form solution and the finite-element solution is observed. Note also that the neutral-surface locations, both Z_{nx}/h and Z_{ny}/h, for the polyester-rubber plate are bounded above by Z_{nx}/h and below by Z_{ny}/h of the aramid-rubber plate.

Figure 5 contains plots of solutions of a single-layer (0°) and a two-layer (0°/90°) square plate ($a = b = 1$, $a/h = 5$, $\Delta t = 0.5$, $q_0 = 1$) of aramid-rubber under suddenly applied impulse loading,

$$q(x,y,t) = q_0 \sin(\frac{\pi x}{a}) \sin(\frac{\pi y}{b}) H(t - 5).$$

where H(t) is the Heavyside step function. Note that the neutral-surface location, Z_{nx}/h, does not reach a steady value like in the case of step loading, but the mean value is about the same as that in the case of step loading.

Thus far, the transverse load was assumed to be sinusoidal (with respect to x and y) to facilitate closed-form solutions. Since the finite element method does not have any limitation on the load distribution, one can use the finite element model developed herein for the analysis of plates subjected to uniformly distributed loads. Figure 6 contains the plots of the transverse deflection, transverse velocity

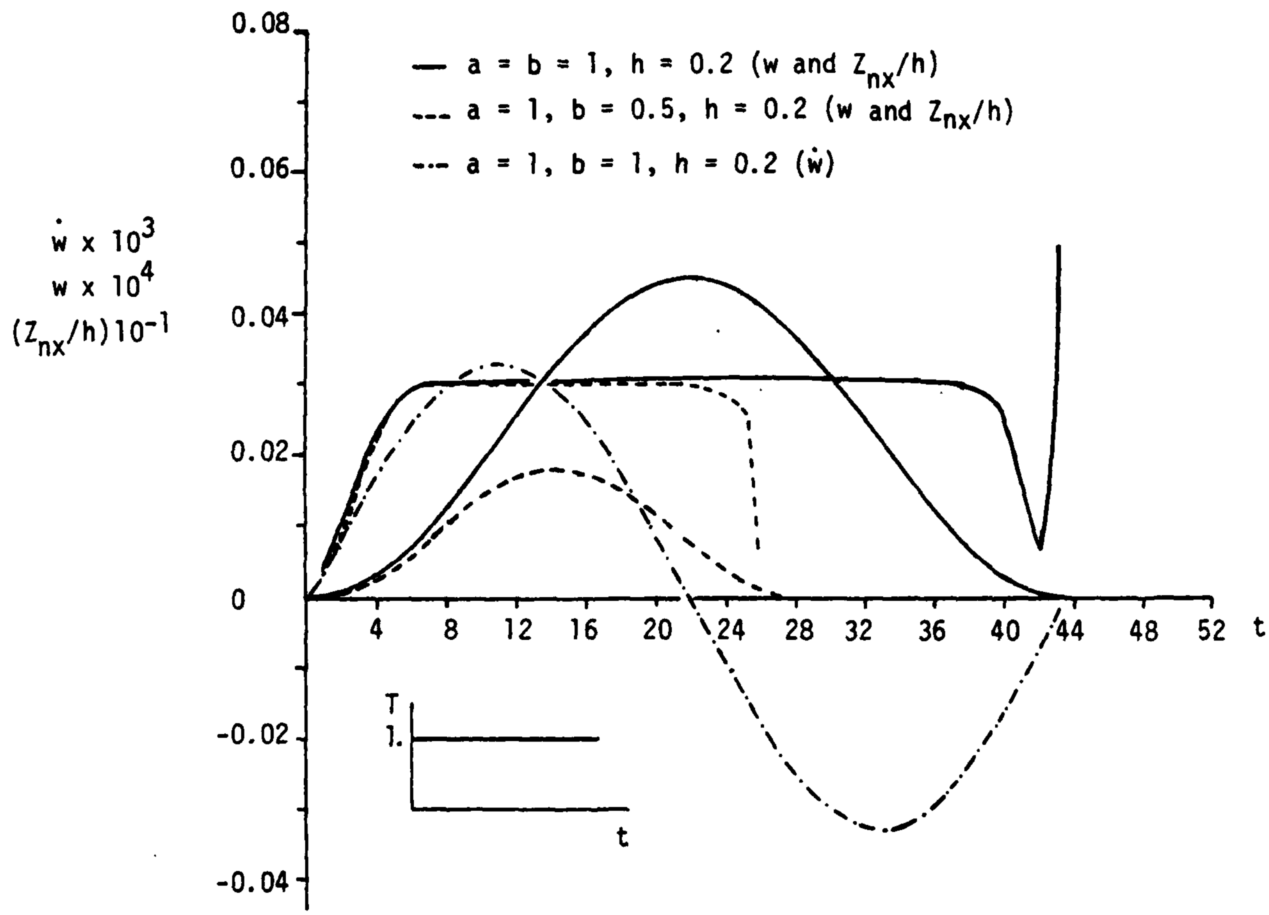

Figure 3. Transient response of a single-layer (0^o), bimodular, rectangular plate of polyester-rubber material under suddenly applied, sinusoidally distributed, step loading.

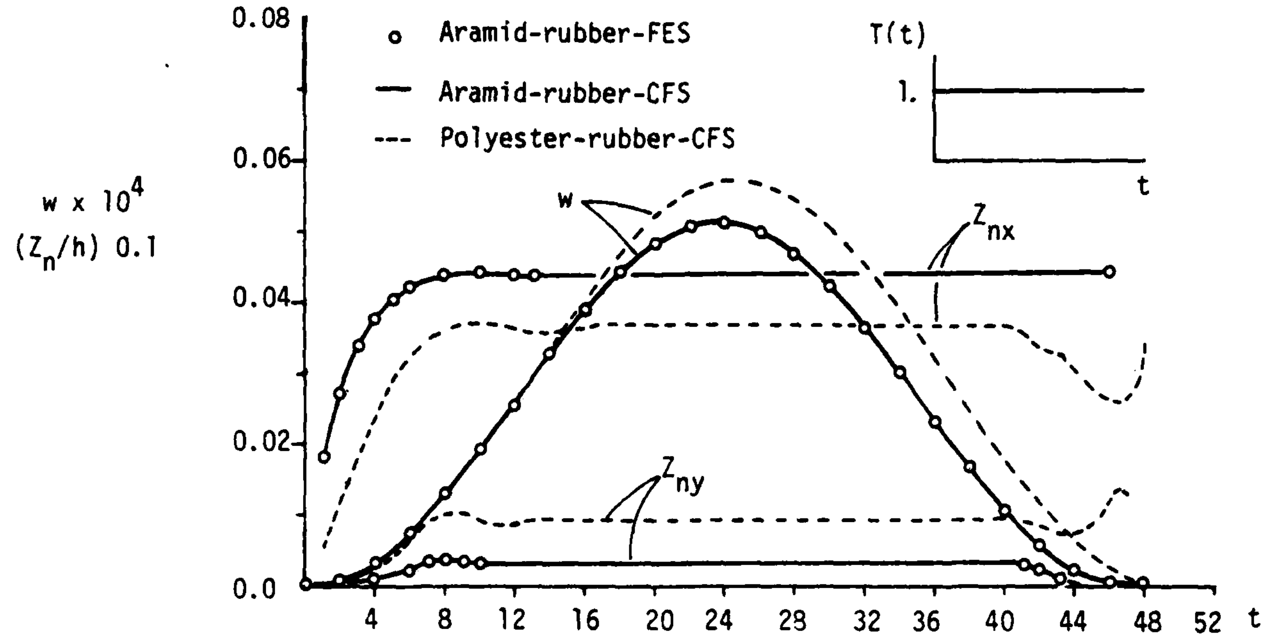

Figure 4. Transient response of two-layer ($0^o/90^o$), bimodular, square plates ($a = b = 1$, $h = 0.2$, $q_0 = 1$, $\Delta t = 0.5$) of aramid-rubber and polyester-rubber materials subjected to suddenly applied, sinusoidally distributed, step loading.

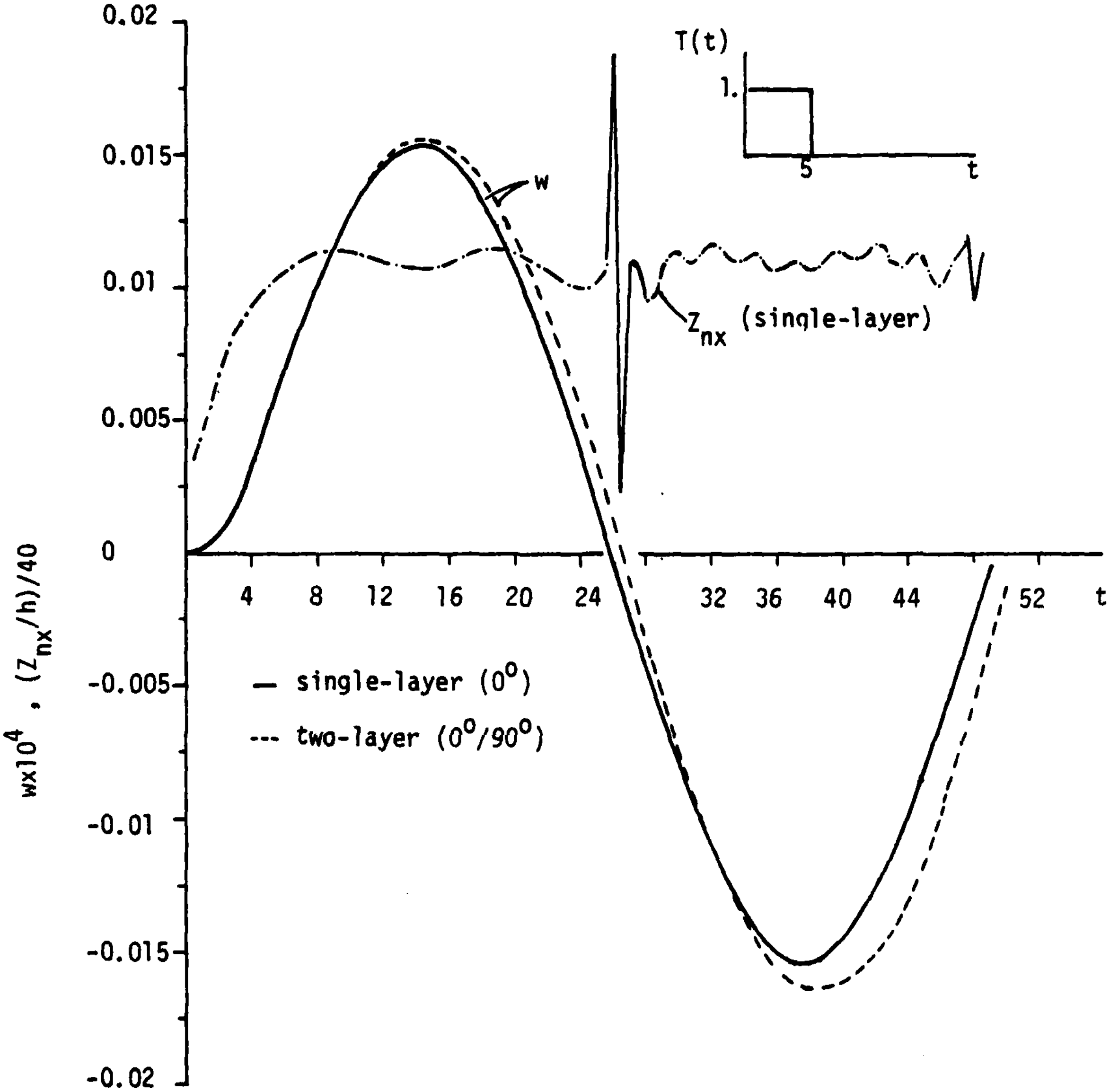

Figure 5. Transient response of single-layer and two-layer, bimodular, square plates of aramid-rubber material ($a = b = 1$, $h = 0.2$, $q_o = 1$, $\Delta t = 0.5$) under suddenly applied, sinusoidally distributed, pulse loading.

and the neutral-surface location, Z_{nx}/h, for a single-layer (0°) and a two-layer (0°/90°) square plate ($a = b = 1$, $a/h = 5$, $\Delta t = 0.5$, $q_0 = 1$) of aramid-rubber under suddenly applied step loading,

$$q(x,y,t) = q_0 H(t).$$

The response curves resemble those of ordinary plates (see [15]). Although the amplitude of the transverse deflection is quite large compared to that of a sinusoidally-loaded plate, the magnitude of the neutral-surface location is about the same. This completes the discussion of the numerical examples.

Summary and Conclusions

The transient analysis of bimodular, composite, rectangular plates is presented. Finite element as well as closed-form solutions are presented for rectangular bimodular plates of aramid-rubber and polyester-rubber materials. The finite element solutions are found to agree very well with the closed-form solutions. From the dynamic (transient) response of the bimodular plates, it is apparent that the shear deformation and plate thickness (irrespective of the transversse shear strains) increase the amplitude and period of the transverse deflection. The results of the present study should be of interest to composite-materials designers and researchers.

Acknowledgment

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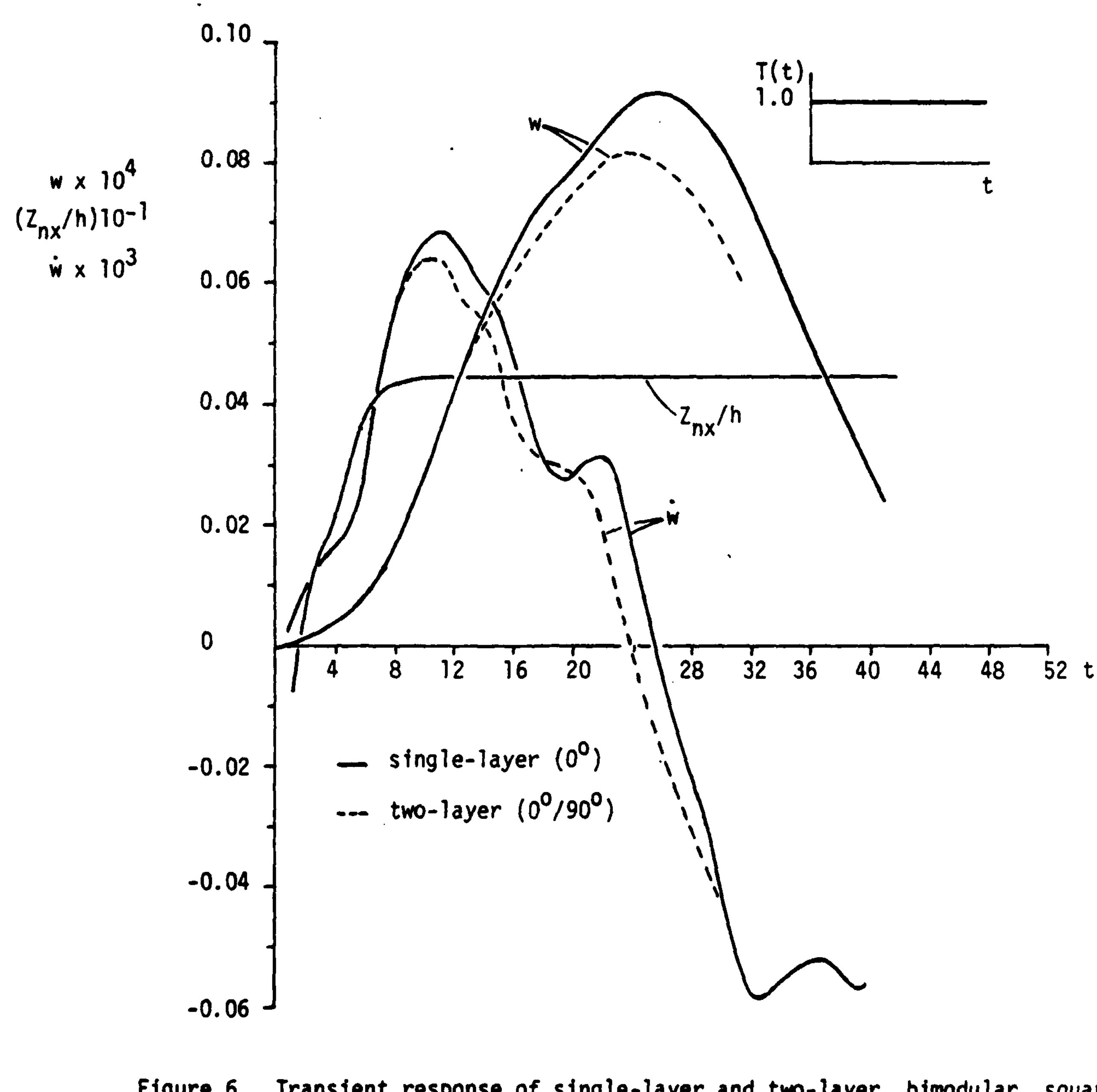

Figure 6. Transient response of single-layer and two-layer, bimodular, square plates ($a = b = 1$, $h = 0.2$, $q_0 = 1$, $\Delta t = 0.05$) of aramid-rubber material under suddenly applied, uniformly distributed, step loading.

References

1. S. Timoshenko, Strength of Materials, Part II: Advanced Theory and Problems, 2nd ed., Van Nostrand, Princeton, NJ, 1941, pp. 362-369.

2. S. A. Ambartsumyan, "The Axisymmetric Problem of a Circular Cylindrical Shell Made of Material with Different Stiffnesses in Tension and Compression," Izvestiya Akademiya Nauk SSSR, Mekhanika, No. 4, 1965, pp. 77-85; English translation, National Technical Information Service Document AD-675312 (1967).

3. N. Kamiya, "Transverse Shear Effect in a Bimodulus Plate," Nuclear Engineering and Design, Vol. 32, No. 3, 1975, pp. 351-357.

4. N. Kamiya, "An Energy Method Applied to Large Elastic Deformation of a Thin Plate of Bimodulus Material," Journal of Structural Mechanics, Vol. 3, No. 3, 1975, pp. 317-329.

5. N. Kamiya, "Thermal Stress in a Bimodulus Thin Plate," Bulletin de l'Academie des Sciences, Serie des Sciences Techniques, Vol. 24, 1976, pp. 365-372.

6. R. M. Jones and H. S. Morgan, "Bending and Extension of Cross-Ply Laminates with Different Moduli in Tension and Compression," Computers and Structures, Vol. 11, No. 3, 1980, pp. 181-190.

7. J. N. Reddy and C. W. Bert, "Analyses of Plates Constructed of Fiber-Reinforced Bimodulus Materials," Mechanics of Bimodulus Materials, C. W. Bert (ed.) AMD Vol. 33, ASME, NY, 1979, pp. 67-83.

8. C. W. Bert, V. S. Reddy, and S. K. Kincannon, "Deflection of Thin Rectangular Plates of Cross-plied Bimodulus Material," Journal of Structural Mechanics, Vol. 8, No. 4, 1980, pp. 347-364.

9. J. N. Reddy and W. C. Chao, "Finite-Element Analysis of Laminated Bimodulus Composite-Material Plates," Computers and Structures, Vol. 12, 1980, pp. 245-251.

10. J. N. Reddy, C. W. Bert, Y. S. Hsu, and Reddy, V. S., "Thermal Bending of Thick Rectangular Plates of Bimodulus Composite Materials," Journal of Mechanical Engineering Science, Vol. 22, No. 6, 1980, pp. 297-304.

11. C. W. Bert, J. N. Reddy, V. S. Reddy, and W. C. Chao, "Bending of Thick Rectangular Plates Laminated of Bimodulus Composite Materials," AIAA Journal, Vol. 19, 1981, pp. 1342-1349.

12. N. M. Newmark, "A Method of Computation for Structural Dynamics," ASCE Journal of the Engineering Mechanics Division, Vol. 85, EM3, 1959, pp. 67-94.

13. J. M. Whitney and N. J. Pagano, "Shear Deformation in Heterogeneous Anisotropic Composite Plates," ASME Journal of Applied Mechanics, Vol. 37, 1970, pp. 1031-1036.

14. C. W. Bert, J. N. Reddy, W. C. Chao, and V. S. Reddy, "Vibration of Thick Rectangular Plates of Bimodulus Composite Material," ASME Journal of Applied Mechanics, Vol. 48, No. 2, 1981, pp. 371-376.

15. J. N. Reddy, "On the Solutions to Forced Motions of Layered Composite Plates," Research Report No. VPI-E-81-24, Department of Engineering Science and Mechanics, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061.

16. T. Y. Tsui and P. Tong, "Stability of Transient Solution of Moderately Thick Plate by Finite Difference Method," AIAA Journal, Vol. 9, 1971, pp. 2062-2063.

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18. SUPPLEMENTARY NOTES

Portions of this paper are accepted for presentation at the Ninth U.S. National Congress of Applied Mechanics, June 21-25, 1982, Cornell University, Ithaca. The paper will appear in its entirety in the Journal of Composite Materials.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Bimodular (bimodulus) materials, composite materials, closed-form solutions, finite-element solutions, laminates, plates, transient analysis, time integration techniques.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Finite-element and closed-form solutions to the equations of motion governing layered composite plates of bimodular materials are presented for rectangular plates with all edges simply supported without in-plane restraint and tangential rotation and subjected to suddenly applied, sinusoidally distributed, transverse loadings. Finite element results are also presented for the same problem but with uniformly distributed step loading. The finite element results are found to be in good agreement with the closed-form solutions.

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2	OU 79-8	Analyses of Plates Constructed of Fiber-Reinforced Bimodulus Materials	J.N. Reddy and C.W. Bert
3	OU 79-9	Finite-Element Analyses of Laminated Composite-Material Plates	J.N. Reddy
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